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# Wind turbine inverter robust loop-shaping control subject to grid interaction effects

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**Abstract**—An  $\mathcal{H}_\infty$  robust control of wind turbine inverters employing an LCL filter is proposed in this paper. The controller dynamics are designed for selective harmonic filtering in an off-shore transmission network subject to parameter perturbations. Parameter uncertainty in the network originates from the grid and the number of wind turbines connected. Power converter based turbines inject harmonic currents, which are attenuated by passive filters. A robust high order active filter controller is proposed to complement the passive filtering. The  $\mathcal{H}_\infty$  design of the control loop enables desired tracking with integral effect while bounding the induced change. The design was tested in an aggregated model of the London Array offshore wind power plant and compared with traditional PI controller designs. Robust stability and performance and a reduction of control effort by 25% are obtained over the full envelope of operation.

**Index Terms**—Inverters, Power conversion harmonics, Power system control, Robust Control, Wind farms.

## I. INTRODUCTION

OFFSHORE wind turbines (WT) are increasing in power rating. Turbine output control requirements have been extended as the power rating has increased. The requirements have been further extended by the move to a power electronic interface. Modern type-4 wind turbines are interfaced to the point of common coupling (PCC) by current-controlled voltage source inverter (VSI) systems [1], [2]. VSIs have fast dynamic response and high quality of the power injection [3]. The current contains components at switching frequency caused by the pulse-width modulation (PWM) switching process [4], [5]. Additionally, the controlled output current is disturbed by polluted PCC voltage due to harmonics of the fundamental power frequency and harmonic resonance in the offshore transmission network [6].

Damping of switching frequency components by L, LC or LCL ac-side filter improves attenuation [7]. L filters require high frequency switching and in high power applications, such as wind turbines, the switching frequency is kept low to reduce losses [8]. The LCL filter provides ideally 60dB per decade harmonic rejection compared with 20dB of the L filter at lower inductance values hence makes lower switching frequencies possible [8]. The LCL filter attenuates the harmonics within the first carrier group, and additional damping of 2<sup>nd</sup> and 3<sup>rd</sup> order harmonics is achieved using trap filters [9].

A challenge in the LCL filter design is the resonance characteristic [10]. Active converter resonance damping was proposed for an LC filter by injecting a damping voltage proportional to the filter capacitance [11]. This virtual resistor control loop was extended to LCL filters to damp resonance peaks [12]. An optimal virtual resistor value was found to be a function of the LCL filter resonance frequency and capacitance [13]. However, impedance seen from the inverter is only partially known at the design state, leading to suboptimal resistor values and decreased damping under uncertain conditions [14]. The sideband harmonics in a resonant grid contains multiples of the power system fundamental and will excite undamped system dynamics [15]. Transmission system uncertainty extends the envelope of parameters, and control must encapsulate both uncertainty and power system harmonics [16].

This can be achieved by using grid voltage feed forward to suppress induced current distortion with proportional-resonant(PR) control. PR control introduces an infinite gain at a selected resonant frequency [17]. Repetitive control (RC) extends the idea to a parallel combination of PI and many resonant controllers [18]. Hybrid designs such as a combination of PR with odd harmonic RC were proposed by [19]. The RC theory was extended to an adaptive solution by [20], and robust RC control wrt. grid frequency change was introduced by [21]. Multiple PI controllers in the  $dq0$  were introduced as a variation of PR in the  $\alpha\beta\gamma$  reference frame in [18]. Hysteresis Band and nonlinear sliding mode control strategies were suggested in [22] and [23].

This paper takes another route by suggesting a control design that combines robust performance and selective filtering. A single repetitive  $\mathcal{H}_\infty$  design is introduced by cascading notch filters in the synthesis. This enables a wide bandwidth of the notch filters without sacrificing performance while guaranteeing stability in the full envelope of operation. A major contribution is the novel idea of ensuring low attenuation of system frequency harmonics to minimize cascade-controller interaction. The network is aggregated and the WTs are a controllable voltage source with harmonic distortion. This enables a study of the interaction between the network, grid and VSI dynamics for varying parameters.

The paper is organized as follows. After a discussion on control requirements, an analytical model is introduced and uncertainty is discussed. Evaluation criteria are then defined and an  $\mathcal{H}_\infty$  controller is designed and compared with the traditional controllers in simulation, followed by conclusions.

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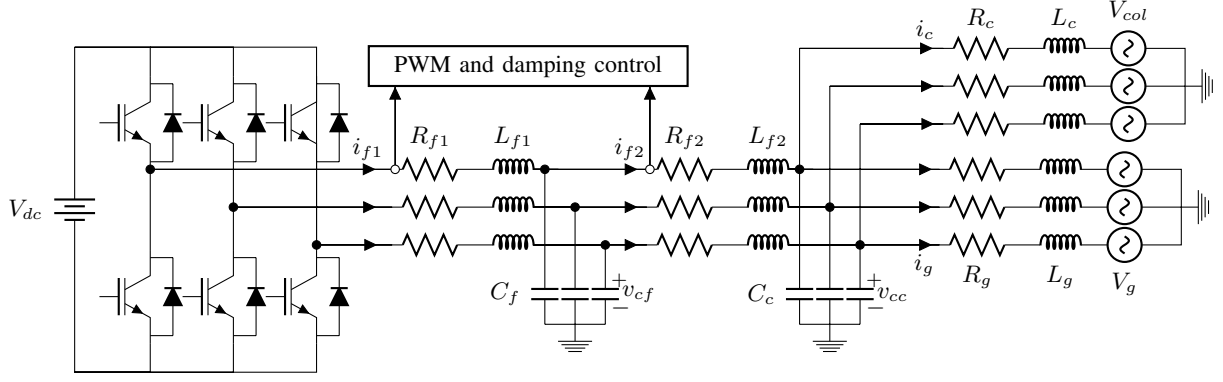


Fig. 3. Three-phase VSI with LCL filter connected to grid and transmission network.

TABLE I  
RELATIVE UNCERTAINTY IN SYSTEM PARAMETERS

Potential	Structure	Element	Uncertainty $\pm$
0.69kV LV	LCL Filter	$C_f, Z_{f1}, Z_{f2}$	15%
33kV HV	Collector Grid	$C_c, L_c, R_c, L_g, R_g$	75%

The aggregated system is uncertain in its parameters. The uncertainties in the collector cables, grid and internal filter are listed in TABLE I. Space vector transformation and measurement of the line voltage enables the transformation of the system equations into Park's d-q frame rotating synchronously with the grid angular speed. Assuming that the mutual capacitance and inductance between the phases are zero, the dynamics of the inverter is transformed to,

$$\begin{aligned}
 L_{f1} \frac{di_{f1}^{dq}}{dt} + R_{f1} i_{f1}^{dq} &= v_{inv}^{dq} - v_{cf}^{dq} - v_d^{dq} + D L_{f1} D \omega i_{f1}^{dq} \\
 L_{f2} \frac{di_{f2}^{dq}}{dt} + R_{f2} i_{f2}^{dq} &= a v_{cc}^{dq} - v_{cf}^{dq} - v_d^{dq} + D L_{f2} D \omega i_{f2}^{dq} \\
 L_c \frac{di_c^{dq}}{dt} + R_c i_c^{dq} &= a^2 v_{cc}^{dq} - a^2 v_{col}^{dq} + D L_c D \omega i_c^{dq} \\
 L_g \frac{di_g^{dq}}{dt} + R_g i_g^{dq} &= a^2 v_g^{dq} - a^2 v_{cc}^{dq} + D L_t D \omega i_g^{dq} \\
 C_f \frac{dv_{cf}^{dq}}{dt} &= i_{f1}^{dq} + i_{f2}^{dq} + D C_f D \omega v_c^{dq} \\
 C_c \frac{dv_{cc}^{dq}}{dt} &= (1/a^2)(i_g^{dq} - i_c^{dq}) - (1/a)i_{f2}^{dq} + D C_c D \omega v_c^{dq},
 \end{aligned} \tag{5}$$

where,

$$\omega = \begin{bmatrix} 0 & \omega_g \\ -\omega_g & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{6}$$

and  $L_{f1}, L_{f2}, L_c, L_g, R_{f1}, R_{f2}, R_c, R_g, C_f$  and  $C_c$  are diagonal matrices with the dimensions  $2 \times 2 \in \mathbb{R}^+$  composed of the nominal values of the components in the dq frame projected to the primary side of the transformer,  $a$  is the pad-mounted transformer ratio given by 0.69kV/33kV and  $v_d^{dq}$  is the potential difference across a virtual damping resistor. The nominal system component values are frequency dependent. This frequency dependence is comprised in the system uncertainty description. The system frequency  $\omega_g$  is assumed to be

constant. The model is bounded by the limits of the physical system given by the switching technology used in the VSI. The injected current is limited by the maximum current rating of the inverter, and the load voltage at the terminals is limited to the maximum output voltage of the inverter. The LCL filter added to the system to attenuate the switching harmonics has an unwanted resonance frequency that should be damped to enable the design of a high-bandwidth current controller and mitigate the grid-induced distortion. The LCL filter resonance can be damped by injecting a damping voltage proportional to the capacitor current in the filter [12]. The injected damping voltage is introduced by defining the damping resistance  $R_d$  and the decoupling voltage,

$$v_d^{dq} = R_d(i_{f1}^{dq} + i_{f2}^{dq}). \tag{7}$$

It should be noted that the use of a directly measured capacitor current could inject noise into the system, and one could use a low-pass filtered signal to overcome this issue. Assuming that the system in (5) is decoupled by feedback and no collector system present, the damping properties of the system wrt. the choice of damping resistor  $R_d$  can be investigated. The nominal value of the resistor is [25],

$$R_d = \frac{1}{3\omega_{res} C_f}. \tag{8}$$

Given that the virtual resistance is calculated from the nominal resonant frequency of the LCL filter, the damping properties of the chosen resistance will change with parametric uncertainty of the grid inductance. Additive uncertainty on  $L_g$  is introduced as,

$$L_g = L_{gn}(1 + \delta). \tag{9}$$

The change of the resonant frequency of the LCL filter as a function of the uncertainty parameter  $\delta$  and the nominal frequency is found to be,

$$\omega_{res}(\delta)^2 = \frac{(L_{gn} + L_{f2})(L_{gn}(1 + \delta) + L_{f1} + L_{f2})}{(L_{gn}(1 + \delta) + L_{f2})(L_{f1} + L_{f2} + L_{gn})} \omega_{res}|_{\delta=0}^2. \tag{10}$$

The virtual resistance was determined from the nominal resonant frequency and its attenuation properties from inverter voltage to grid current  $H(s)$ , will change as

$$\left| \frac{H(\omega_{res})|_{\delta=\delta}}{H(\omega_{res})|_{\delta=0}} \right| = \frac{L_{gn} + L_t}{L_{gn}(1 + \delta) + L_{f1} + L_{f2}}. \tag{11}$$

Equation (11) shows that the attenuation of the resonant peak at the perturbed resonant frequency will decrease with increased  $\delta$ . The magnitude of the change is a function of the relation between the inverter side inductance and the grid side inductance. Minimization of costs opts for lower values of  $L_{f1}$  which then increases the effect of a change in the uncertain parameter  $L_g$ , which determines the control requirements needed for operation in an uncertain grid.

#### A. Uncertain Continuous Time State-Space Model

External voltage disturbances and uncertainty in the system parameters wrt. both cross coupling and active damping requires an uncertain formulation of the system dynamics. The uncertainty of parameters should be included in the system model in addition to possible unmodelled internal dynamics and time delays. The system can be formulated as a nominal system with dynamic disturbances and perturbations of the parameters.

$$\begin{aligned} \frac{dx(t)}{dt} &= A_0x(t) + B_0u(t) + E_0d(t) + W_0x(t) \\ y(t) &= Cx(t), \end{aligned} \quad (12)$$

where

$$\begin{aligned} x(t) &= \begin{bmatrix} i_{f1}^{dq} & i_{f2}^{dq} & i_c^{dq} & i_g^{dq} & v_{cf}^{dq} & i_{cc}^{dq} \end{bmatrix}^T \\ u(t) &= \begin{bmatrix} v_{inv}^{dq} \end{bmatrix}^T, \quad d(t) = \begin{bmatrix} v_{col}^{dq} & v_g^{dq} \end{bmatrix}, \end{aligned} \quad (13)$$

with  $C = I$  and  $A_0, B_0, E_0$  and  $W_0$  as shown in (14). The uncertainty is represented by the uncertain time delay of the actuator and parametric uncertainty on each of the electrical elements of the system listed in TABLE I. The general parameter  $\alpha$  is with added uncertainty given as  $\alpha = \alpha_0 + \delta_\alpha$ , which results in the uncertain state-space representation,

$$\begin{aligned} \frac{dx(t)}{dt} &= (A_0 + A_u)x(t) + (B_0 + B_u)u(t) \\ &\quad + (E_0 + E_u)d(t) + W_0x(t) \\ u_u(t) &= F(s)u(t), \quad y(t) = Cx(t), \end{aligned} \quad (15)$$

where the subscript 0 denotes the nominal system parameters, and  $u$  is the uncertain perturbations.  $F(s)$  is the actuator transfer function matrix and is included as an uncertain time delay. The set of possible plants is illustrated in Fig. 4. The linear system with uncertainty in (15) can be rewritten as two identical decoupled SISO systems with multiplicative uncertainty in the Laplace domain by introducing frequency dependent uncertainty regions. Component uncertainty with respect to values and frequency dependency is contained in the uncertainty regions. The output of such system with uncertainty weight  $W_O(s)$  and  $0 \leq \Delta \leq 1$  is given as,

$$y(s) = \begin{bmatrix} (1 + W_O(s)\Delta)G_{if2v_{inv}}(s) \\ G_{if2v_g}(s) \\ G_{if2v_{col}}(s) \end{bmatrix}^T \begin{bmatrix} v_{inv}(s) \\ v_g(s) \\ v_{col}(s) \end{bmatrix}, \quad (16)$$

where  $\Delta = 1$  would represent 100% uncertainty. Omitting the Laplace operator, the closed-loop system output is,

$$y = \underbrace{\frac{GK}{I+GK}}_T r + \underbrace{\frac{1}{I+GK}}_S G_{ad} - \underbrace{\frac{GK}{I+GK}}_T n \quad (17)$$

where  $G = (1 + W_O(s)\Delta)G_{if2}$  and  $K$  is the controller. System theory states that  $S+T = 1$ , which shows that ideal fulfillment of objective one in section II would provide perfect noise rejection. The system setup is further described in section IV-D.

#### IV. VSI CONTROLLER DESIGN

The PI controller is the best candidate for regulating DC values with zero steady state error due to its infinite DC gain. PR control and RC are not able to guarantee a closed-loop unity attenuation of power system harmonics. The PI control is therefore used as comparison to the  $\mathcal{H}_\infty$  controller. The filter capacitor can be neglected with the approximation that the LCL-filter converges to a simple L filter at low frequencies [8]. Generalized parameter estimation techniques such as symmetrical optimum (SO) or magnitude optimum (MO) are traditionally used for control design [31]. Requirements for the control system are bandwidth and overshoot [32]. Internal mode control (IMC) is considered as a closed loop systematic approach for parameter specification [33]. A major disadvantage of the PI control design is the inability to shape the disturbance rejection loop at the harmonic frequencies. The PI control law for the VSI system is given by,

$$u(t) = L_\Sigma \omega_g \begin{bmatrix} i_{f2}^d \\ i_{f2}^q \end{bmatrix} + K_p e_{f2}^{dq} + K_i \int e_{f2}^{dq} dt, \quad (18)$$

where the error signal  $e_{f2}^{dq} = r_{f2}^{dq} - i_{f2}^{dq}$  and

$$L_\Sigma = L_{f1} + L_{f2} + aL_g. \quad (19)$$

$K_p$  and  $K_i$  are found for SO, MO and IMC given equal performance criteria defined in section II.

##### A. Magnitude optimum

Low order plants without time delay and one dominant time constant is often tuned using the magnitude optimum criteria. For a general second order system,

$$G(s) = (1 + \tau_1 s) / (1 + \sigma s) K^{-1} \quad (20)$$

where  $\sigma$  is the sum of parasitic time constants smaller than  $\tau_1$ , it has the solution [34],

$$K_i = \frac{1}{2K} \left( \frac{1}{\sigma} + \frac{1}{\tau_1} - \frac{1}{\tau_1 + \sigma} \right), \quad K_p = \frac{1}{2K} \left( \frac{\tau_1}{\sigma} + \frac{\sigma}{\tau_1} \right) \quad (21)$$

which if  $\sigma \ll T_1$  provides full cancellation of the plant pole by the controller zero.

##### B. Symmetrical optimum

The *symmetrical optimum* method aims at shaping both disturbance rejection and reference tracking by optimizing their common characteristic equation. The loop transfer function is shaped to be in the form  $\omega_0^2(2s + \omega_0)/s^2(s + 2\omega_0)$ . A system in the form of (20) with  $\tau_1 \gg \sigma$  has the solution [35],

$$K_p = \frac{\tau_1}{\alpha K \sigma}, \quad K_i = \frac{\tau_1}{\alpha^3 K \sigma^2} \quad (22)$$

with typically  $2 \leq \alpha \leq 3$  [36]. The closed-loop response of the system (22) has a large overshoot and a pre-filter is usually designed to shape the reference input [37].



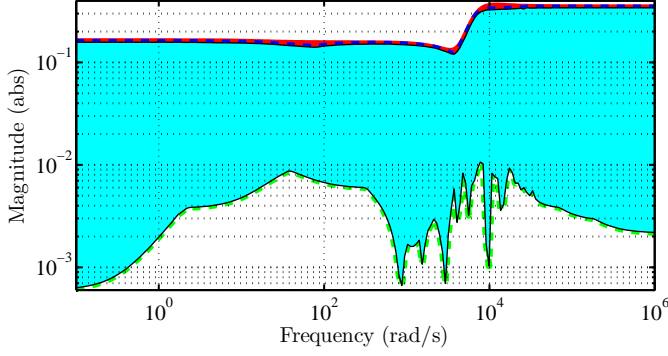


Fig. 6. Relative error  $L_O$  of  $G_{II}$  and uncertainty filter  $W_O$  with  $|\Delta| < 1$  for 200 samples of  $G_{II}$  representing the relative uncertainty as  $G_{II} = (I + W_o\Delta)G_0$ . Weighting filter  $W_o(s)$  (—), worst case uncertainty (stipple), best case uncertainty (dastdot), relative error area (shaded).

with the matrix  $P(s)$

$$P(s) = \begin{bmatrix} 0 & 0 & 0 & 0 & H_{if2v_i} W_o \\ W_p & W_p G_{if2v_c} & W_p G_{if2v_g} & -W_p & W_p G_{if2v_i} \\ 0 & 0 & 0 & 0 & W_u \\ W_t & 0 & 0 & 0 & W_t G_{if2v_i} \\ -I & -G_{if2v_c} & -G_{if2v_g} & 1 & -G_{if2v_i} \end{bmatrix}. \quad (28)$$

The  $\mathcal{H}_\infty$  controller design opts to minimize the  $\mathcal{H}_\infty$ -norm of the lower fractional transformation of  $P$  and  $K$ ,

$$N = F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}, \quad (29)$$

which is the transfer function matrix from exogenous signals  $w$  to performance signals  $z$ . The uncertainty is included as an internal exogenous input,  $w_i$ , and a performance signal  $z_i$ . The external exogenous signals included are the current reference  $i_{f2}^*$ , the grid disturbance  $v_g$  and the network disturbance  $v_c$ . The disturbance transfer function matrix  $G_{dw}$  maps the exogenous signals to the output current  $i_{f2}$ . The disturbance dynamics are specified in the  $P(s)$  matrix and provides the relation between the exogenous inputs  $w$  and the performance signals  $z$ . The complex uncertainty description  $W_o$  is lumped using a third order filter that covers the set of plants in  $G_{II}$  as shown in Fig. 6. It is possible to include neglected series and parallel resonances and specific frequency dependent components in the uncertainty description by increasing the order of  $W_o$ . Higher order filter results in a higher order controller and a conservative design. The parametric uncertainty is assumed to be sufficient to cover the frequency dependency within the controller bandwidth.

### E. Weighting functions

The weighting functions specify the relative weight on signal frequency characteristics and should reflect the design criteria set in section II.

$W_p$  specifies the tracking performance of the system by bounding the sensitivity below the wanted bandwidth,  $\omega_{bi}$ , of the system and decrease at high frequencies. The high-pass filter specifies a minimum bandwidth and approximate integral

TABLE II  
WEIGHTING FUNCTION PARAMETERS

$W_p$		$W_t$		$W_u$	
$\tau_i$	$5e^{-3}$	$A_s$	1.65	$g_u$	1/9
$A_p$	2	$A_t$	1		
$g_{p1}$	1.60	$g_{t1}$	7.12		
$g_{p2}$	1.44	$g_{t2}$	7.64		
$g_{p3}$	0.72	$g_{t3}$	7.93		
$g_{p4}$	0.01	$g_{t4}$	7.51		
$\omega_{pl}$	20 Hz	$\omega_{tl}$	400 Hz	$\omega_{uh}$	3900 Hz
$\omega_{ph}$	140 Hz	$\omega_{th}$	1000 Hz	$\omega_{ul}$	470 Hz

action. The performance weight is found as,

$$W_p(s) = \frac{s/A_p + \omega_{ph}}{s + \omega_{pl}\tau_i} \prod_{k=1}^n \left( 1 / \left( \frac{s^2 + 2kg_p\omega_{bn}\omega_0s + (\omega_0k)^2}{s^2 + 2k\omega_{bn}\omega_0s + (\omega_0k)^2} \right) \right), \quad (30)$$

where  $\omega_{bn}$  is the bandwidth of the non-ideal notch filter,  $g_k$  is the gain of the  $k$ 'th notch filter and  $\tau_i$  provides approximate integral action. The performance will be dictated by  $W_p$  for  $\omega < \omega_{bi}$  and by  $W_t$  for  $\omega > \omega_{bi}$  ensuring fulfillment of the second and third requirement. The notch filters are inversely included in the selection of  $W_p$  to satisfy (33).

The control signal weight,  $W_u$ , is designed to allow for sufficient control effort while realizing the fourth requirement.  $W_u$  is implemented as a second order weight to increase roll off,

$$W_u(s) = \frac{(s/g_u^{1/2} + \omega_h)^2}{(s + \omega_l)^2}, \quad (31)$$

where  $g_u$  is a tuning parameter to select the attenuation of the filter.

The weighting function on the complementary sensitivity function  $W_t$  is designed for low attenuation in a narrow band around the  $n$  significant harmonics to fulfill the first requirement. It is the upper bound on  $T$ , and is additionally shaped for high-frequency roll-off and force  $T \rightarrow \epsilon$  for all  $\omega \in \omega_h$ ,

$$W_t(s) = \frac{s/A_t + \omega_{th}}{s + \omega_{tl}} \prod_{k=1}^n \left( \frac{s^2 + 2kg_t\omega_{bn}\omega_0s + (\omega_0k)^2}{s^2 + 2k\omega_{bn}\omega_0s + (\omega_0k)^2} \right). \quad (32)$$

The parameters are listed in TABLE II. Picking  $A_p > 1$  provides room for  $S > 1$  when  $u(s)/r(s) > 1$ , and is necessary to comply with the robust performance bound,

$$\|W_p(s)S(s)\|_\infty + \|W_o(s)T(s)\|_\infty < 1 \quad (33)$$

and the robust stability bound,

$$\|T(s)\|_\infty < \|1/W_o(s)\|_\infty. \quad (34)$$

The closed-loop system includes modes associated with the notch filters and thus have an oscillatory impulse response with frequency and damping selected in  $W_t$ . The controller specification designates a 1Hz bandwidth of the notch filter which equates a notch filter damping of  $\zeta = 0.001$ . Excitation of notch filter dynamics when tracking the current reference



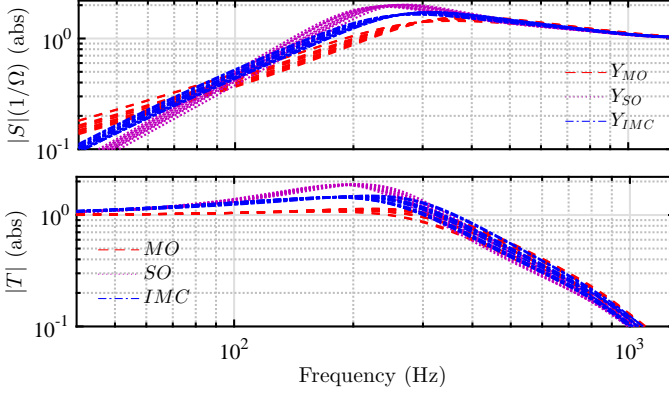


Fig. 7. **a)** Variation of perturbation bodeplot of closed loop harmonic admittance ( $|S|$ ) at terminals. MO shows least integral action while IMC and SO are comparable. **b)** Bode plot of closed-loop  $T$ . The closed loop performance of the PI loop shaping controllers achieves  $|T| > -3dB$  for  $\omega < \omega_{bi}$ , but both IMC and SO have resonance peaks  $|T| > 1$  which leads to overshoot.

is avoided by inclusion of filter  $C(s)$  on the reference signal  $r(s)$ ,

$$r(s)^* = C(s)r(s) = \left( \prod_{k=1}^n \left( \frac{z_n}{p_n} \right) \right) r(s), \quad (35)$$

where  $z_n$  and  $p_n$  are the zero and pole pair of the  $n$ 'th notch filter in the closed loop, identified by their damping properties. In addition to designing a prefilter, the controller  $K$  is reduced from a 77 order design to a 14 order using Hankel optimal model reduction while conserving the notch filter dynamics.

## V. PERFORMANCE EVALUATION

The closed loop sensitivity function with uncertainty is illustrated in Fig. 7. The Figure shows the integral action and limitations of the PI design as the closed-loop system disturbance rejection cannot be shaped at the power system harmonics. The complementary sensitivity function  $T = 1 - S$  obtains a bandwidth of  $0.4kHz$  as seen in Fig. 7, but cannot guarantee nominal performance for all system perturbations. To assess the performance and quality in regards to the specifications, quantitative metrics  $J_{d1}$  and  $J_{d2}$  are introduced to measure the control effort and sensitivity attenuation at  $\omega_h$ . With  $S_{v_g}$  being the spectrum of the harmonics,

$$J_{d1}(\omega_h) = \frac{1}{2} \int_{\omega=\omega_h-\omega_d}^{\omega_h+\omega_d} (|H_{i_{v_{inv}v_g}}(\omega)|^2) S_{v_g} d\omega \quad (36)$$

$$J_{d2}(\omega_h) = \frac{1}{2} \int_{\omega=\omega_h-\omega_d}^{\omega_h+\omega_d} (|1 - |H_{i_{f_2v_g}}(\omega)||^2) S_{v_g} d\omega.$$

The normalized worst case gain of the system to output disturbance with respect to both output current and inverter switching is shown in Fig. 8. The duality of the measured metrics and the limitation of the first order compensator with respect to disturbance attenuation and control system limits is clear, as no PI-controller is significantly better at the selected harmonics.

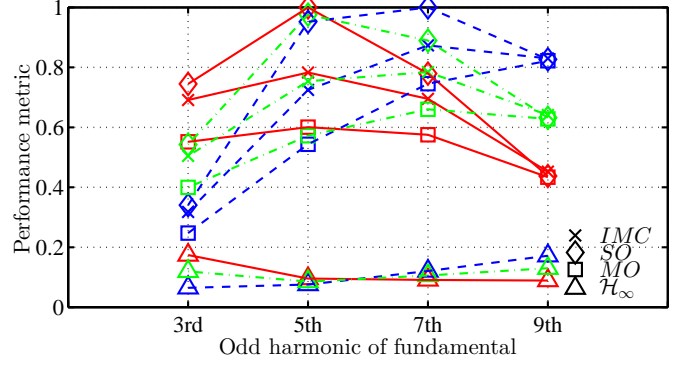


Fig. 8. Normalized  $J_d(\omega_h)/|(J_d(\omega_h))|_\infty$ , performance metric  $J_d(\omega_h)$  for IMC, SO, MO and  $\mathcal{H}_\infty$  controllers. Disturbance to control input  $J_{d1}$  (solid), sensitivity  $J_{d2}$  (stipple) and combined  $J_d$  (dashdot).

## VI. SIMULATION

The system is evaluated using the performance metric from (36) under worst case uncertainty criteria. The  $\mathcal{H}_\infty$  optimal design guarantees internal stability of the closed-loop system when  $F_l(P, K) < 1 \forall \omega$ . The robust performance and stability is shown in Fig. 10 and the system is stable and performing to specifications for all plants in  $G_{II}$ . The challenge of tuning the system using notch filters is evident due to the simultaneous increase in amplitude of  $W_p$  and  $W_t$ . Time domain simulations are performed in an SimPower Systems model of the London Array wind park using an (N+1) model shown in Fig. 9. Tracking results are shown in Fig. 11a for a doublet step reference change, and the system obtains a rise-time of  $1.3ms$  which corresponds to a bandwidth of  $0.27kHz$  which is lower than the specification. The rise-time is a consequence of the conservatism introduced and having no overshoot, which results in an improved settling time compared to the PI-controllers. The control effort associated with the reference tracking is shown in Fig. 11b and the  $\mathcal{H}_\infty$  controller has 25% less effort compared to the best PI control.

Fig. 12 shows the output response to a disturbance on grid voltage and measurements containing power system harmonics. In Fig. 12a, the controller ensures almost unity sensitivity for the harmonic frequencies and complements the designed filters compared to the phase-lag and amplification of the best PI-controller. Fig. 12b shows an average attenuation of 90% from noise to output. The fact that noise is rejected when sensitivity is forced towards unity is a fundamental property of control theory and shows how this design methodology can be used to complement the effect of existing filters.

The performance metric from section V for the  $\mathcal{H}_\infty$  design in Fig. 8 shows the notch filter design efficiency in obtaining a closed-loop sensitivity  $\epsilon \in [0.8; 1]$ .  $J_d$  is close to 0.1 for all  $\omega_h \pm \omega_d$  and improving  $J_{d1}$  and  $J_{d2}$  by minimum 75%.

Robust stability and performance is demonstrated by the multiplicative uncertainty implemented. Uncertainty caused by frequency dependence of components could be represented by using an uncertainty extending throughout a wide range of frequencies. Any additional uncertainty or neglected dynamics could be obtained by using a weighting function of higher



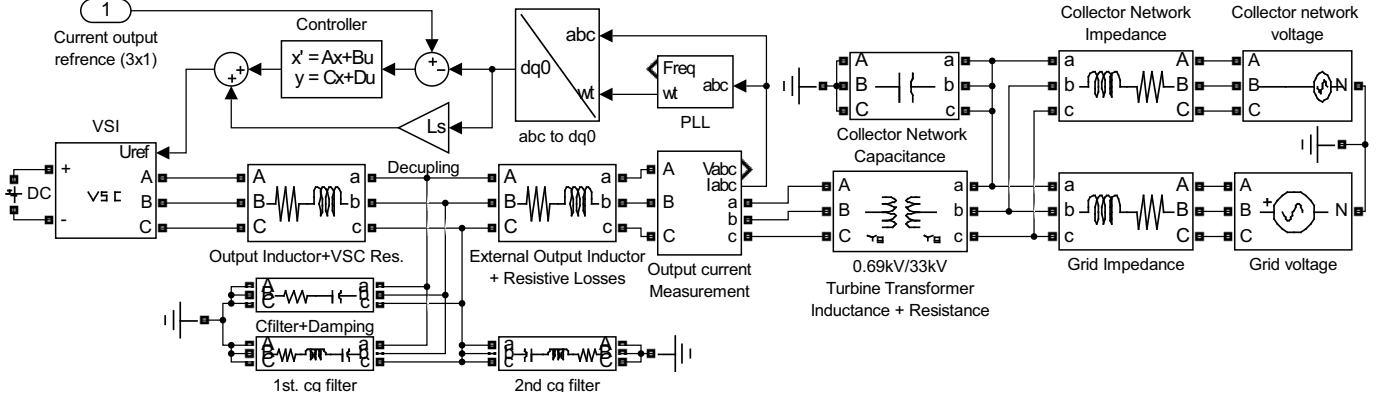


Fig. 9. N+1 model of London Array wind park with generic controller implementation and constant VSI DC-link voltage. The system is balanced and the PLL synchronized with grid voltage vector. Carrier group trap filters are included and tuned to 1st and 2nd harmonic of the switching frequency. Weak PCC/Grid system with an SCR of 20 to emphasize collector network parameter perturbation.

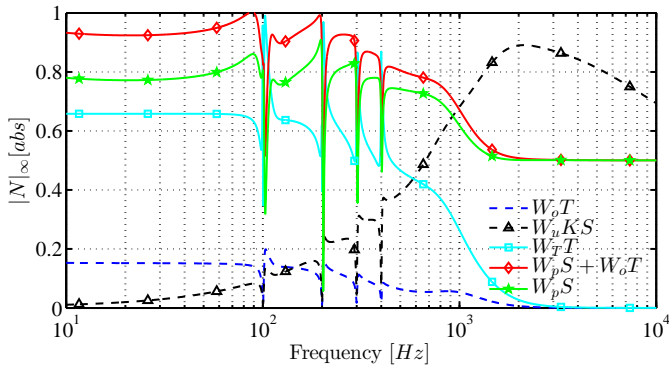


Fig. 10.  $\mathcal{H}_\infty$  closed loop worst case singular values.  $|T_{zw}|_\infty < 1$  is obtained and conforms with robust performance (33) and robust stability (34) criteria.

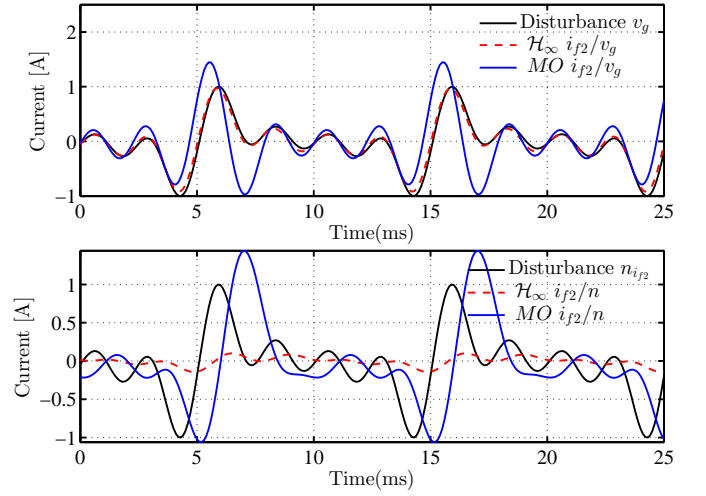


Fig. 12. Disturbance  $v_g$  and  $i_{f2}$  measurement noise composed of normalized harmonic signal,  $\omega = [100\text{Hz}, 200\text{Hz}, 300\text{Hz}, 400\text{Hz}]$  a) Disturbance attenuation. Output current,  $\mathcal{H}_\infty$  compared to best PI controller for harmonic grid disturbance.  $\mathcal{H}_\infty$  shows compliance within specifications and ensures a sensitivity of unity, while the PI-controller amplifies the disturbance. b) Noise rejection. The controller attenuates power system harmonic noise on the transformer current measurements used in feedback according to the notch filter dynamics.

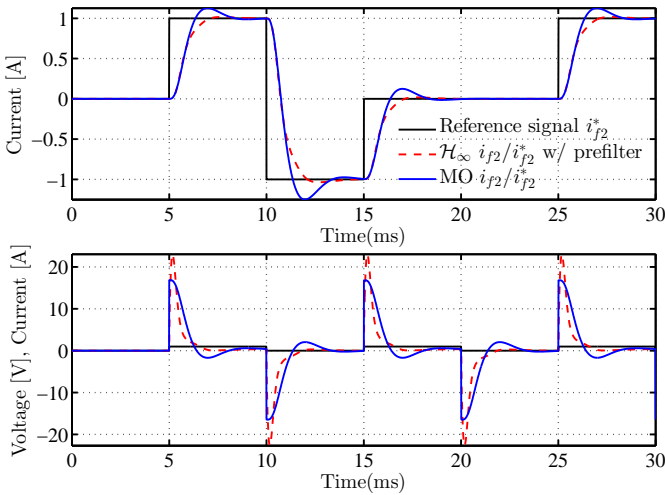


Fig. 11. Reference tracking. a) Output current,  $\mathcal{H}_\infty$  compared to best PI controller.  $\mathcal{H}_\infty$  shows faster tracking with no overshoot and within specifications. b) Control effort.  $\mathcal{H}_\infty$  has a larger initial effort but avoids overshoot and obtains a 25% reduction compared to PI.

order [38]. In systems with low frequency series and parallel grid resonances, a more aggressive choice of uncertainty region could be used to ensure robust stability. Such change would affect the controller synthesis towards a conservative design and by an increase of controller order. The extension of the uncertainty region is decided from the uncertainty in wind-park layout. Variations of cable length, placement, specifications and grid strength all contribute to the combined uncertainty. For wind-farms where the relative uncertainty is known, the fixed parameter robust controller instantaneously provides its designed characteristics while additional algorithms in the loop and an adaptation transient is present for adaptive control. If the performance cannot be achieved by

robust control, adaptive control should be considered [39].

A high order controller in a real-world discrete control structure puts requirements on the computing power needed. Modern WTs are equipped with powerful processors, fiber cables for communication and high frequency measurement equipment. In practice the probable challenges that can arise in implementation are associated with the effects of unmodelled external systems such as the PLL.

## VII. CONCLUSION

Inverter control is a challenging part of designing the output stage of a type-4 wind turbine. In this paper, traditional loop shaping methods were compared to an  $\mathcal{H}_\infty$  optimal design with focus on minimizing disturbance rejection for use in inner loop system current control schemes. The  $\mathcal{H}_\infty$  design using notch filters tuned at the odd harmonic frequencies combined with approximate integral action showed a considerable improvement in performance concerning overshoot, control effort and specific output disturbance rejection. Robust performance and stability was achieved with a uncertainty span equivalent to a collector system with one to multiple connected turbines.

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